Transitions in (1+1) light front ϕ^4 theory using quantum computing method

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Abstract

We study the phase transitions in (1+1) ϕ^4 theory in the light-front frame using both classical computing and quantum computing methods. The transition at the ground state from a single-particle dominant state to a three-particle dominant state can be revealed by the parton distribution function and indicated by crossing of mass square eigenvalues at a strong coupling. We discuss quantum computing as a method of exploring quantum phase transition in the light-front (1+1) ϕ^4 theory.

I. INTRODUCTION

Topological phase transition [1] in quantum field theory is an emerging collective phenomenon that can be explored by non-perturbative methods. However, the computational resources needed often grow exponentially with increasing resolution. Quantum computing is promising in reducing the memory consumption, as N configurations can be encoded by only $log_2 N$ qubits in compact encoding [2]. Although the fully functional scalable fault-tolerant quantum computer is not available yet, we are entering an important era in quantum computing commonly known as the Noisy Intermediate-Scale Quantum (NISQ) era [3]. Instead of waiting the hardware to get mature, it is important to investigate NISQ era opportunities with, perhaps, up to a few hundred qubits and to develop simulation packages that address physics problems.

Problems that are hard to solve on classical computers might be easy to solve on quantum computers. Feynman [4], in 1982, pointed out that because nature itself is quantum mechanical, quantum computing will allow us to do the "exact" simulation. There is also a great need in simulating systems with high-entanglement [5]. For example, hadronic matter subjected to large quantum fluctuations cannot be described analytically. Lattice QCD has achieved numerous success in numerically computing quantities in these systems, such as the masses of hadrons. However, the Monte Carlo simulations of lattice QCD often face the obstacle known as the "sign problem" and are unable to simulate the real time dynamics of high-energy collisions between hadrons. Therefore we need a better language to describe the

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highly-entangled systems. Quantum computers make use of interference and entanglement, so it is a direction that is worth pursuing as a new pathway to overcome this difficulty.

Advances in quantum computing architecture and algorithms have convinced us quantum computing can out-perform classical computing in some problems where classical computing algorithms have hit their barriers. For instance, Shor's algorithm [6] can be used to efficiently find the prime factors of a large composite integer. Experiments [7, 8] show that quantum computers are able to sample from probability distributions faster than the most advanced classical computer.

The specific problem that I'd like to address in this paper is using quantum simulation to solve a conceptually simple theory, the two-dimensional ϕ^4 theory. Studying the two-dimensional ϕ^4 theory is meaningful because there is a rich physics known in this theory, such as the topological phase transitions. Currently only a few studies on this subject using quantum computing methods, among which [9] studies symmetry protected topological phases and transitions of infinite one-dimensional spin-$\frac{1}{2}$ chains using variational quantum eigensolvers. It is very interesting to study the phase transitions using quantum computing methods because they occur in a strong correlation domain and thus hard to study using classical computing.

II. METHODOLOGY

Currently, there are mainly two methods for solving the eigenvalues problems with quantum computing techniques. One is the quantum phase estimation (QPE) algorithm, which offers exponential speedup over classical methods [10]. The other is the Variational Quantum Eigensolver (VQE) quantum algorithm, which requires less coherence time [11] compared to the QPE.

We start with the VQE algorithm using Qiskit [12] package developed by IBM. The VQE is based on the variational principle to solve the ground state of a system. In the classical variational method, the expectation value of the Hamiltonian is evaluated using a trial wavefunction. While in the quantum variational method, the Hamiltonian is converted to a Pauli-string matrix and evaluated using a quantum circuit conveniently called "ansatz". The Pauli-string is a string of ordered Pauli matrices or the identity matrix, corresponding to the operations on the qubits in "x", "y", "z" directions or no operation on a real quantum

device. The ansatz contains a number of parameters that can be tuned. Classical optimizers are used for optimizing the parameters of the ansatz in each iteration, until the expectation value of the Hamiltonian is minimized. Then the ansatz with the optimal set of parameters will represent the ground state. Physical observables can be evaluated using this final ansatz to study the ground state properties. Fig. 1 is an example of a "TwoLocal" ansatz with 16 parameters. What the ansatz does is to rotate the initial state $Q_0 = \{q_0, q_1, q_2, \cdots\}$ to a new quantum state.

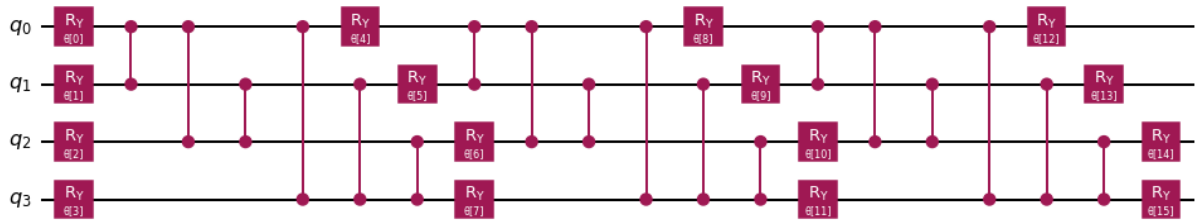

FIG. 1. A "TwoLocal" ansatz with 4 qubits and 3 repetition layers has 16 parameters.

It turns out that there is no standard rule for how to choose the best ansatz and how many parameters should we set. Various candidates for an ansatz have been proposed by the quantum computing community, including layered gate ansatz [13], alternating operator ansatz [14] and tensor network ansatz [15]. There are also physics motivated ansatzes such as the coupled cluster ansatz [16]. The number of parameters needed depend on many factors, such as the symmetry properties and the dimension of the Hamiltonian. One needs to perform a series of tests to determine the working ansatz. There is also a question of how to set the initial state. A well-chosen initial state will result in faster convergence for the iteration. A review of the recent development of the VQE method can be found in [17].

To apply the VQE method to quantum field theories, we first consider one of the simplest interacting quantum field theories, the (1+1) (one space dimension plus one time dimension) ϕ^4 theory:

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{\mu^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4. \tag{II.1}$$

For the VQE approach, we need to choose a suitable classical optimizer. There is a large set of optimizers in the market such as the gradient-based optimizers L-BFGS-B [18] and SLSQP [19], the derivative-free optimizers COBYLA [20] and Nelder-Mead [21]. This step will involve efforts to find the optimizer that does not get trapped in the local minimums.

And it is hard to predict which optimizer will eventually give better results, for example, the optimizer that does not perform very well in noise-free simulations might do better in noisy simulations. We can design a short-subproject to study how the performance of each available optimizer changes with increasing noise level of the simulation. Apart from that, one can also use the quantum annealing techniques to find the minimum [22]. After reaching the variational minimum of the expectation value of the Hamiltonian with satisfactory precision, the ansatz can be used to evaluate interesting physical quantities such as the parton distribution function which will reveal the transition of the single particle state to multi-particle comoving states in the strong coupling region [23].

III. SIMULATION PROCESS

We start from the (1+1) light front ϕ^4 theory Lagrangian (in symmetric phase for example) [24, 25]

$$\mathcal{L} = \frac{1}{2}\partial^+\phi\partial^-\phi - \frac{\mu^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4, \tag{III.1}$$

and discretize the field in the domain of $-L < x^- < L$ ($n = 1, 2, \cdots$ for periodic boundary condition neglecting the zero mode)

$$\phi = \frac{1}{\sqrt{4\pi}}\sum_n \frac{1}{\sqrt{n}}(a_n e^{-i\frac{n\pi}{L}x^-} + a_n^\dagger e^{i\frac{n\pi}{L}x^-}). \tag{III.2}$$

The resulting Hamiltonian is

$$\begin{aligned} H =& \mu^2 \sum_n^K \frac{1}{n} a_n^\dagger a_n \\ &+ \frac{\lambda}{4\pi}\left(\sum_{k\le l, m\le n}^K \frac{1}{N_{kl}}\frac{1}{N_{mn}}\frac{a_k^\dagger a_l^\dagger a_m a_n}{\sqrt{klmn}}\delta_{m+n,k+l} + \sum_{k,l\le m\le n}^K \frac{1}{N_{lmn}}\frac{a_k^\dagger a_l a_m a_n + a_n^\dagger a_m^\dagger a_l^\dagger a_k}{\sqrt{klmn}}\delta_{k,m+n+l}\right) \end{aligned} \tag{III.3}$$

where $N_{kl} = 1, k \neq l$; $N_{kl} = 2!, k = l$. And $N_{lmn} = 1, l \neq m \neq n$; $N_{lmn} = 2!, l = m \neq n$ or $l \neq m = n$; $N_{lmn} = 3!, l = m = n$. K is the total momentum that controls the resolution of

the discretization.

We solve for the lowest eigenvalue of $M^2 = KH$ using the Variational Quantum Eigensolver (VQE) quantum algorithm in Qiskit [12] developed by IBM. Classical optimizers are used for optimizing the parameters of the ansatze in each iteration. Physical observables can be evaluated with the final optimal ansatze. We prepare a circuit "TwoLocal" with 16 parameters as an ansatze for $K = 9$ odd sector (Hamiltonian matrix of 16×16 dimension encoded by 4 qubits). It is crucial of choosing a suitable optimization method which gives a satisfactory performance.

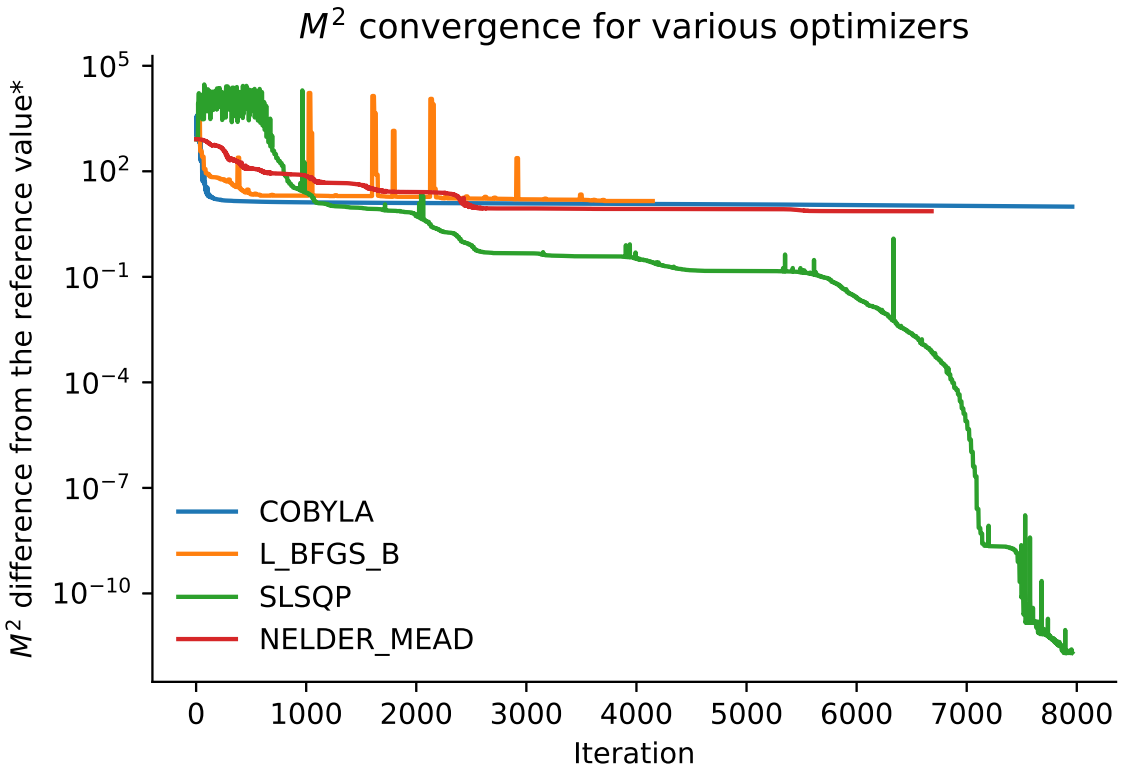

FIG. 2. M^2 convergence for various optimizers for ideal simulations. COBYLA and Nelder-Mead are derivative-free optimization methods. L-BFGS-B uses the solutions and gradients from the most recent iterations to estimate the Hessian matrix. SLSQP is a local gradient-based optimization method using sequential least squares programming. (*The reference value is calculated by classical computing.)

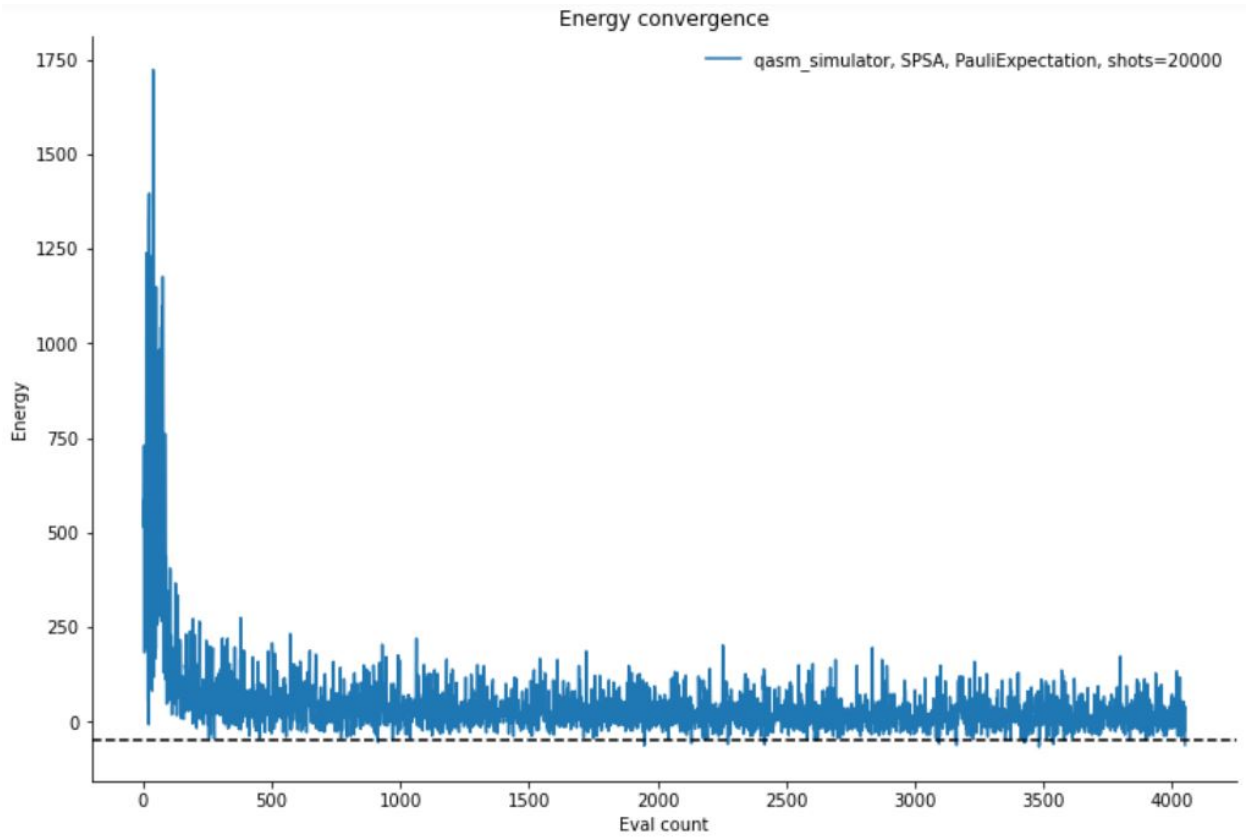

FIG. 3. M^2 convergence for SPSA optimizer for qasm simulation.

IV. THE PARTON DISTRIBUTION FUNCTION

Parton distribution function (PDF) $s(x)$ represents the number density of a particle carrying momentum fraction x. In discretized version,

$$s_i(x_k) = \sum_j |\alpha_{ji}|^2 \frac{\langle \phi_j | a_k^+ a_k | \phi_j \rangle}{N_j}, \tag{IV.1}$$

where N_j is the number of particles in j-th configuration $|\phi_j\rangle$ and $x_k = k/K$. One can check that $\sum_k s_i(x_k) = 1$.

If the eigenvalues m_i^2 are in ascending order, then $i = 1$ corresponds to the lowest eigenvalue. Thus the ground state PDF is

$$s_1(x_k) = \sum_j |\alpha_{j1}|^2 \frac{\langle \phi_j | a_k^+ a_k | \phi_j \rangle}{N_j}. \tag{IV.2}$$

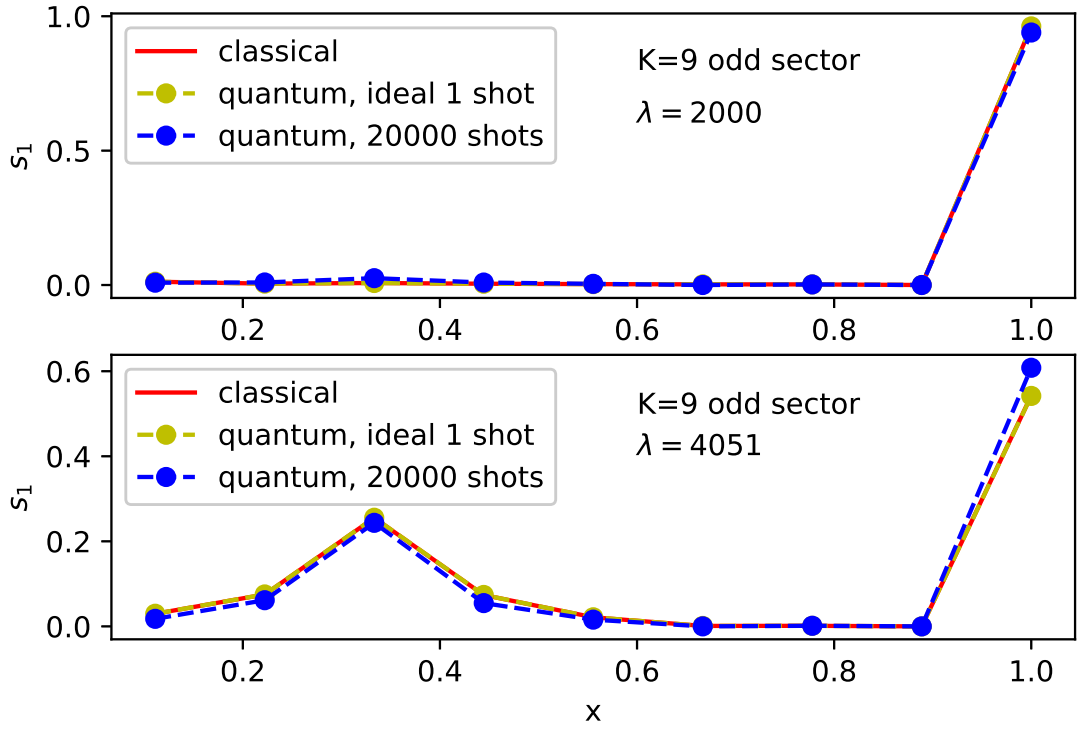

FIG. 4. PDF of the $K = 9$ odd sector ground state of (1+1) light front ϕ^4 theory. We can see a second peak at $x \sim 0.33$ at the stronger coupling, indicating a significant portion of comoving three particles at the odd sector ground state.

V. CONCLUSION AND DISCUSSION

We show that the VQE quantum algorithm can be used to explore the transition properties of the (1+1) light front ϕ^4 theory. To scale up the quantum simulation to higher dimensional matrices, there will be a significant increase in the simulation time. One of the solution is to use parallel quantum simulation, such as the one in [26]. In the NISQ era, we also need to validate our method by noisy simulations. This will involve error correcting codes [27]. The cost of different resources for the quantum computation for a specific problem should also be estimated. These resources include the number of gates, the number of qubits, the number of iterations, the energy cost, and perhaps circuit depth if the

computation is highly parallelizable.

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